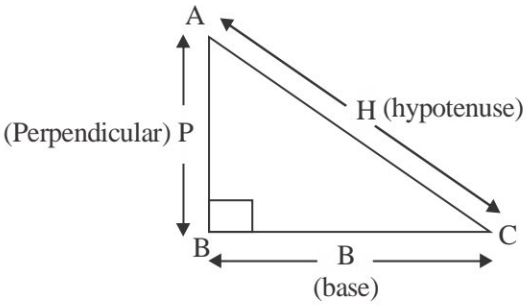
CHAPTER

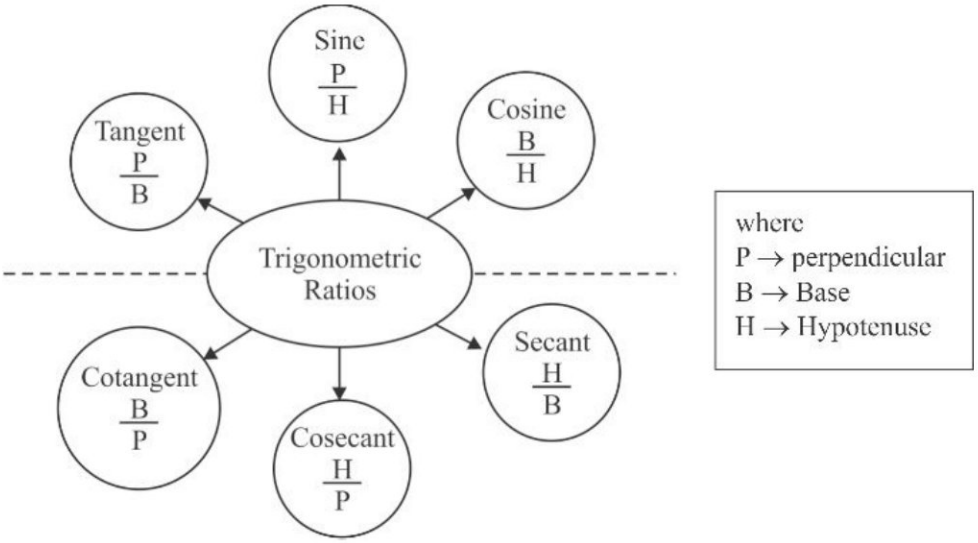
8 Introduction to Trigonometry

KEY POINTS



- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangle.
  Note : For $\angle A$, Perpendicular is BC and base is AB.
  For $\angle C$, Perpendicualr is AB and Base is BC.

Trigonometric Ratios of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

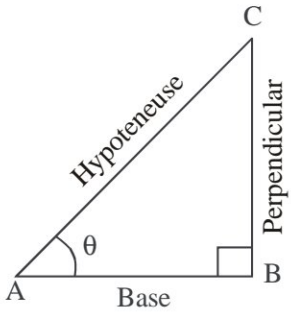


Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentence.

Some **P**eople **H**ave **C**urly **B**rown **H**air **T**hrough **P**roper **B**rushing

$\sin A = \frac{P}{H}$ $\cos A = \frac{B}{H}$ $\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Reciprocal ratios:

$$\sin\theta = \frac{1}{\text{cosec }\theta}, \quad \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

4. Identities

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. Trigonometric ratios of some specific angles

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\sin\theta = \cos\theta$, find the value of θ

2. Find the value of $\tan^4\theta + \cot^4\theta$, if $\sin\theta - \cos\theta = 0$

3. Find the value of $\tan\theta + \cot\theta$, if $\tan^2\theta - 3\tan\theta + 1 = 0$

4. If $\tan\theta = \frac{4}{3}$ then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

5. If $3x = \operatorname{cosec}\theta$ and $\frac{3}{x} = \cot\theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

6. If $x = a\sin\theta$ and $y = a\cos\theta$ then find the value of $x^2 + y^2$

7. If $\cos A = \frac{3}{5}$, find the value of $4 + 4\tan^2 A$

8. Find the value of $9\sec^2 A - 9\tan^2 A$

9. Express $\sec\theta$ in terms of $\cot\theta$

10. If $x = a\sec\theta$, $y = b\tan\theta$, then find the value of $b^2x^2 - a^2y^2$.

11. Find the value of $\frac{1+\tan^2\theta}{1+\cot^2\theta}$, if $\tan\theta=\frac{4}{3}$.

12. Find the value of $\frac{1+\tan^2\theta}{1+\cot^2\theta}$

13. Given $\tan\theta=\frac{1}{\sqrt{3}}$, find the value of $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}$. (CBSE, 2010)

14. If $\sqrt{3}\cot^2\theta-4\cot\theta+\sqrt{3}=0$, then find the value of $\tan^2\theta+\cot^2\theta$.

15. If $5\tan\theta-4=0$, then value of $\frac{5\sin\theta-4\cos\theta}{5\sin\theta+4\cos\theta}$ is

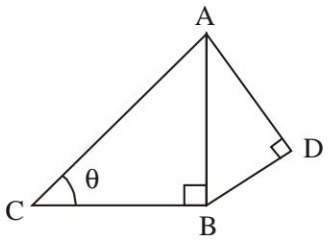
(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{6}$

16. $3\tan^2\theta-3\sec^2\theta+4$ is equal to

(a) 3 (b) 2

(c) 1 (d) 0

17. In Fig. if $AD=4$ cm, BD = 3 cm and CB = 12 cm. then $\cot\theta=$



(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

(c) $\frac{13}{12}$ (d) $\frac{12}{13}$

18. If $x=3\sin\theta+4\cos\theta$ and $y=3\cos\theta-4\sin\theta$ then x^2+y^2 is

(a) 25 (b) 45 (c) 7 (d) 49

19. If $\sin\theta = \frac{a}{b}$, then the value of $\sec\theta + \tan\theta$ is

(a) $\sqrt{\frac{a+b}{a-b}}$ (b) $\frac{a+b}{a-b}$ (c) $\sqrt{\frac{b+a}{b-a}}$ (d) $\frac{b+a}{b-a}$

SHORT ANSWER TYPE QUESTIONS (1)

Prove that :

20. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

21. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \text{Sec}\,\theta$

22. If $x = p\sec\theta + q\tan\theta$ & $y = p\tan\theta + q\sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$

23. If $7\sin^2\theta + 3\cos^2\theta = 4$ then show that $\tan\theta = \frac{1}{\sqrt{3}}$

24. Find the value of $\cos\theta$, if $\sec\theta + \tan\theta = 5$

25. If $3\cot A = 4$, find the value of $\frac{\text{cosec}^2 A + 1}{\text{cosec}^2 A - 1}$.

26. Find the value of $\tan^3\theta + \cot^3\theta$, if $\tan\theta + \cot\theta = 2$.

27. Find the value of $\tan\theta$, if $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$.

(CBSE 2011)

28. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

29. Show that : $\frac{1-\sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

30. Find the value of θ, if $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, $\theta \le 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

31. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

32. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta + 1$

(CBSE 2019, 2023)

33. $\sec A (1 - \sin A) (\sec A + \tan A) = 1$ **(CBSE 2023)**

34. If $\sec \theta = x + \frac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$

35. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

36. Prove that $\cos \theta = \frac{p^2 - 1}{p^2 + 1}$, if $p = \operatorname{cosec} \theta + \cot \theta$.

37. Show that: $x^2 + y^2 + z^2 = r^2$, if $x = r \cos \alpha \sin \beta$, $y = r \cos \alpha \cos \beta$ and $z = r \sin \alpha$

38. Find the value of $\sin^{10} \theta + \operatorname{cosec}^{19} \theta$, if $\sin \theta + \operatorname{cosec} \theta = 2$.

39. Prove that: $2 \sec^2 x - \sec^4 x - 2 \operatorname{cosec}^2 x + \operatorname{cosec}^4 x = \cot^4 x - \tan^4 x$

40. Find the value of $\operatorname{cosec} \theta$, *if* $\operatorname{cosec} \theta - \cot \theta = \frac{1}{3}$

41. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

42. Evaulate : $\frac{\tan^2 60° + 4 \cos^2 45° + 3 \sec^2 30° + 5 \cos^2 90°}{\operatorname{cosec} 30° + \sec 60° - \cot^2 30°}$

43. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$ (CBSE, 2023)
Prove that : $a^2 + b^2 = m^2 + n^2$

LONG ANSWER TYPE QUESTIONS

Prove That:

44. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

45. $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

46. $(1+\cot A+\tan A)(\sin A-\cos A)=\sin A\tan A-\cot A\cos A$

47. If $\sin\theta+\cos\theta=m$ and $\sec\theta+\operatorname{cosec}\theta=n$ then show that $n(m^2-1)=2m$

48. Prove that: $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\operatorname{cosec}\theta$ **(CBSE 2023)**

49. Prove that :

$$\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot\theta}$$

50. If $\frac{\cos\alpha}{\cos\beta}=m$ and $\frac{\cos\alpha}{\sin\beta}=n$, then prove that $(m^2+n^2)\cos^2\beta=n^2$

51. Prove that :

$$\sec^2\theta-\frac{\sin^2\theta-2\sin^4\theta}{2\cos^4\theta-\cos^2\theta}=1$$

52. Prove that : $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

53. Prove that: $\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\frac{\sin\theta}{1-\cos\theta}$

54. If $\sin\theta+\cos\theta=\sqrt{3}$, then prove that $\tan\theta+\cot\theta=1$ **(CBSE 2020)**

55. Prove $\frac{\cot A-\cos A}{\cot A+\cos A}=\sec^2A+\tan^2A-2\sec A\tan A$ **(CBSE 2020 Basic)**

56. Prove $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$ **(CBSE 2020 Basic)**

57. If $\cos(A+B) = \sin(A-B) = \frac{1}{2}$, $0 < A+B < 90°$ and $A > B$ then find the value of A and B. **(CBSE 2020 Basic)**

58. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$. **(CBSE 2020 Standard)**

59. Prove that : $l^2m^2(l^2+m^2+3) = 1$

If $l = \text{cosec}\, x - \sin x, m = \sec x - \cos x$ **(CBSE 2020 Standard)**

60. Prove $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$ **(CBSE 2020 Standard)**

61. Prove that $\frac{(1+\sin x-\cos x)^2}{(1+\sin x+\cos x)^2} = \frac{1-\cos x}{1+\cos x}$ **(CBSE 2019)**

62. Prove that $\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}$ **(CBSE 2019)**

63. If $4\tan\theta = 3$ then find the value of $\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}$ **(CBSE 2018)**

64. Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \sec\theta+\tan\theta$ **(CBSE 2018)**

65. Prove that $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta} + \frac{1}{1+\text{cosec}^2\theta} = 2$

66. Prove that $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta\,\text{cosec}\theta - 2\sin\theta\cos\theta$

67. If $\text{cosec}\,\theta = 4x + \frac{1}{16x}$, prove that $\text{cosec}\theta \pm \cot\theta = 8x$ or $\frac{1}{8x}$